Aesop: White-Box Best-First Proof Search for Lean

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White-Box and Black-Box Proof Search

Proof Search Without Metavariables

Proof Search With Metavariables

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Proof Search With Metavariables

Interactive Theorem Proving Is Great

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Interactive Theorem Proving Is Annoying

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$$a \in [b] \leftrightarrow a = b$$

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$$a \notin xs \to a \notin ys \to a \notin xs ++ ys$$

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$$\mathrm{map}\ f\ xs \subseteq \mathrm{map}\ f\ ys \leftrightarrow xs \subseteq ys \quad \text{if } f \text{ is injective}$$

How Do We Make It Less Annoying?

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1. Convince lots of mathematicians that they should formalise boring stuff for the greater good.
2. Let the computer do the boring stuff.

White-Box and Black-Box Proof Search

Black-box
hammers
SMT solvers
ML-based provers
Coq `sauto`
Agsy

White-box
Coq `eauto`
Matita `auto`
Isabelle `auto`
Isabelle `auto2`
PVS `grind`
ACL2 waterfall
Aesop

White-Box and Black-Box Proof Search

Black-box	**White-box**
fully automatic	needs configuration
powerful	weak
the future	boring old tech

White-Box and Black-Box Proof Search

Black-box	**White-box**
fully automatic	needs configuration
powerful	weak
the future	boring old tech
complex	simple
unpredictable	predictable
opaque	transparent
fixed performance	customisable performance
proof export is hard	proof export is easy(ish)

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Proof Search With Metavariables

Basic Procedure

$$\vdash A \to C \to A \land (B \lor C)$$

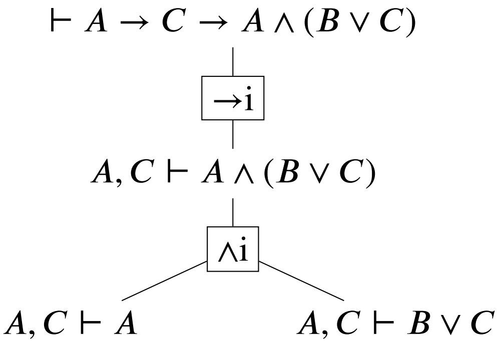
Basic Procedure

$$\vdash A \to C \to A \land (B \lor C)$$

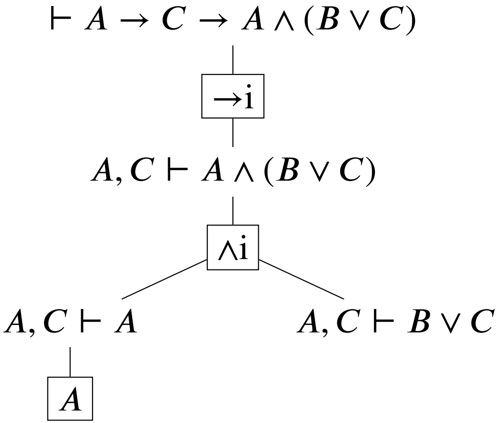
$\to$i

$$A, C \vdash A \land (B \lor C)$$

Basic Procedure



Basic Procedure



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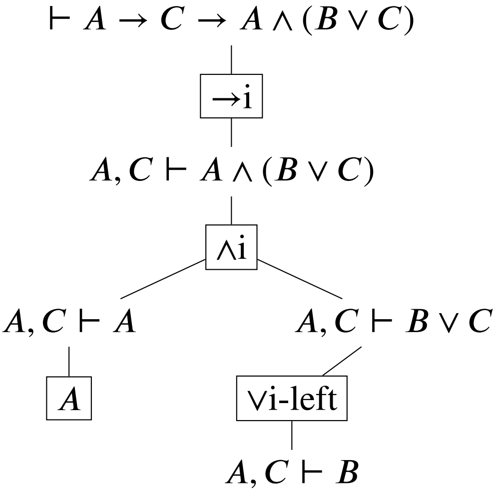

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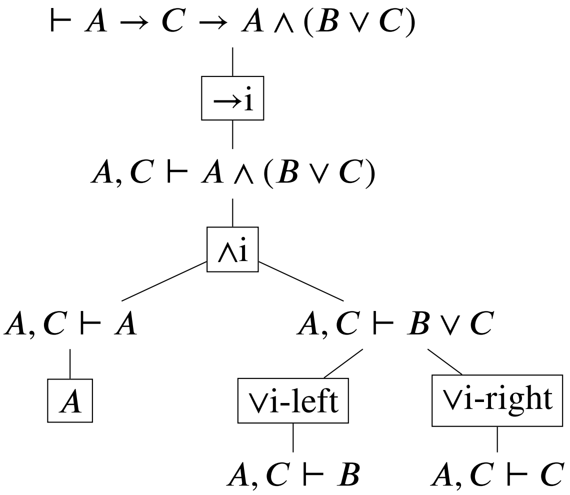
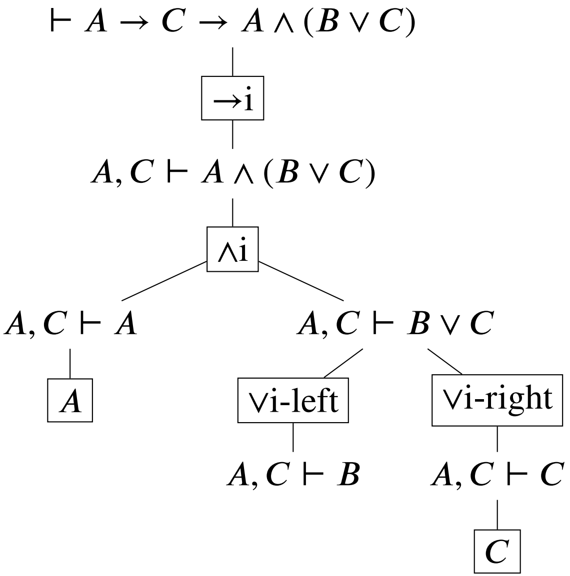
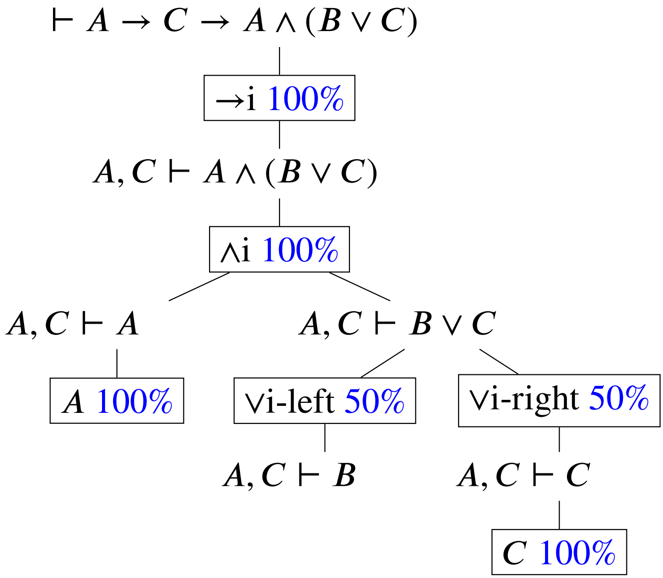


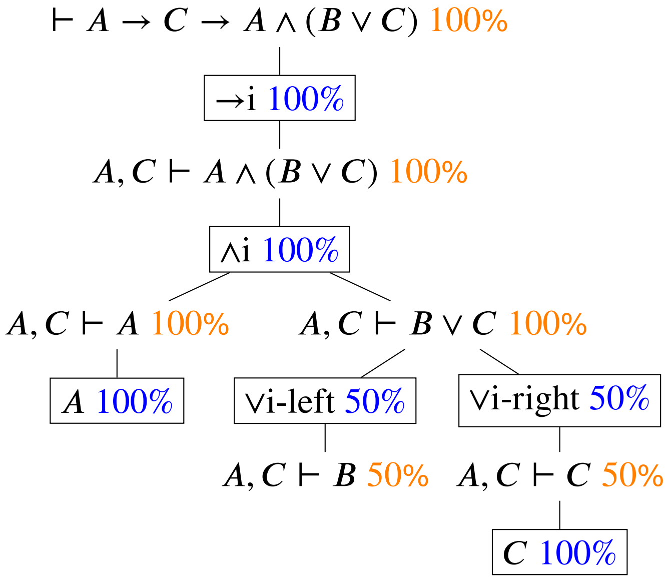
Basic Procedure



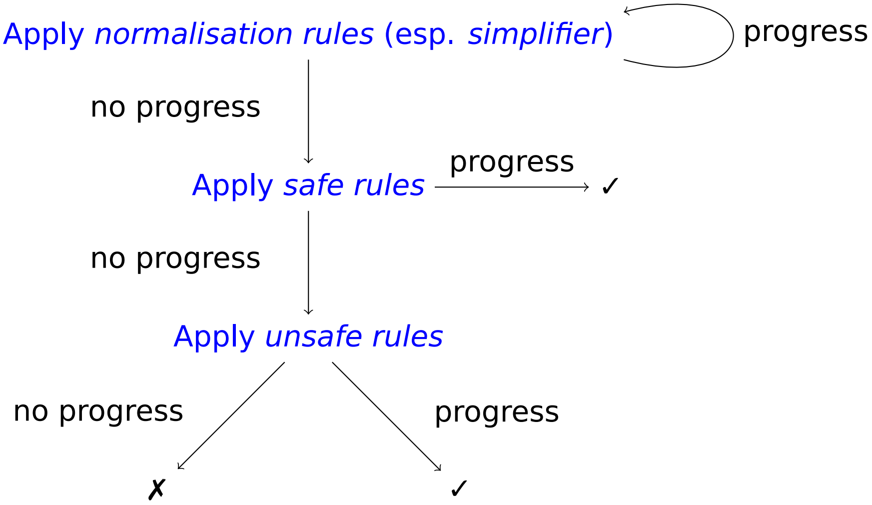
Best-First Search



Best-First Search

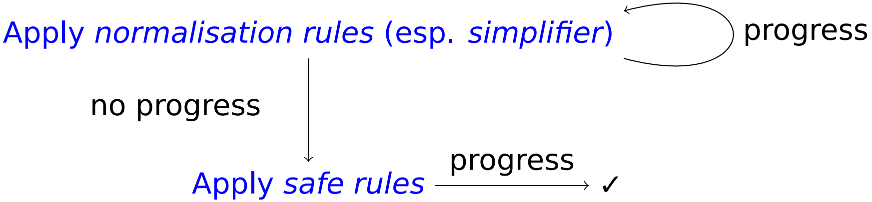


Extensions of the Basic Procedure

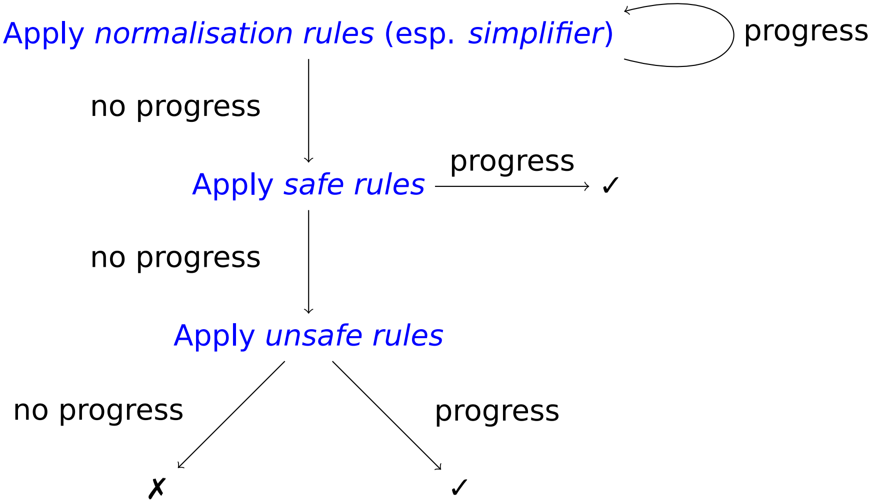


Apply *normalisation rules* (esp. *simplifier*) ↺ progress

$$\frac{\Gamma,\ A,\ B \vdash T}{\Gamma,\ A \wedge B \vdash T}\ \wedge\text{e}$$



$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \land B}\ \land\mathrm{i}$$


Apply *normalisation rules* (esp. *simplifier*)
progress
no progress
Apply *safe rules*
progress
✓
no progress
Apply *unsafe rules*
no progress
progress
✗
✓

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$$a < b,\ a < c,\ b < z \vdash a < z$$

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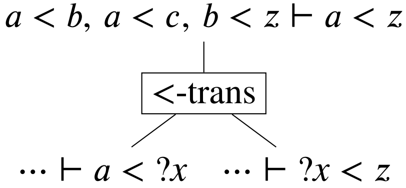
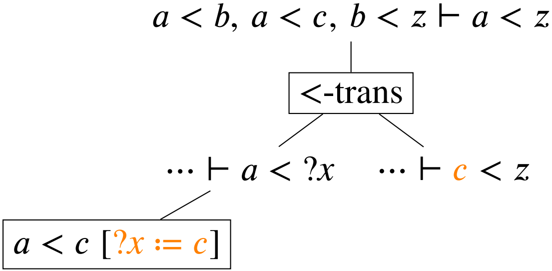
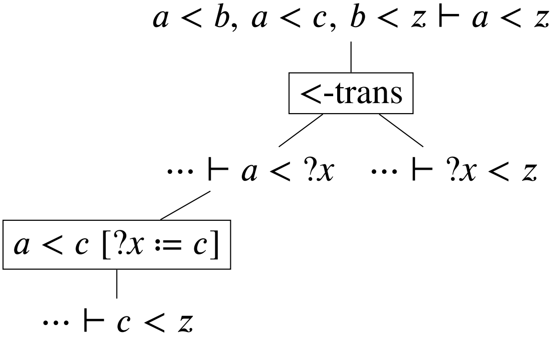


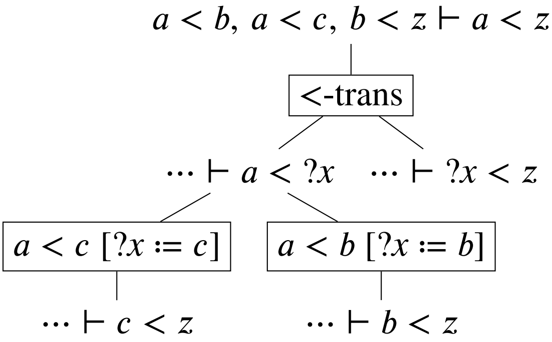
Proof Search With Metavariables



Proof Search With Metavariables



Proof Search With Metavariables



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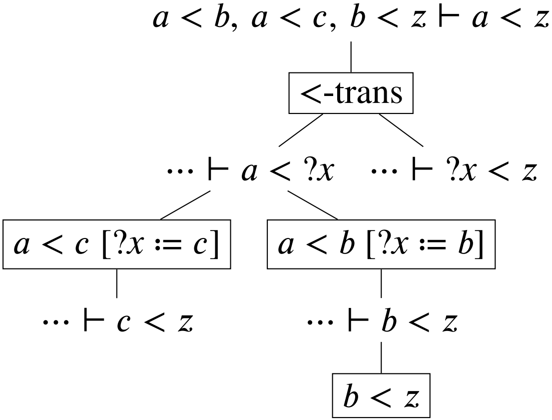

Also in the Paper

- Built-in rules
- Rule indexing
- UI for adding common sorts of rules
- Case studies
  - 173 basic lemmas about lists (Aesop + induction proves 94%)
  - Sequent calculus prover

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